EFFECT OF A MOBILE LOAD ON A NONLINEARLY COMPRESSED STRIP WITH A RIGID FOUNDATION

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The two-dimensional stationary problem of the effect of a mobile load on a nonlinearly compressed strip with a rigid foundation is studied. The case of linear loading and unloading of the medium was analyzed in [1, 2]. In this work, unlike [1, 2], the wave process is studied taking into account the nonlinear loading of the strip material, the effect of the inelastic properties of the medium on the distribution of kinematic parameters and stresses in it is studied, and the form of the surface of the front of the wave reflected from the rigid foundation is determined.

Let a monotonically decreasing normal load move with constant velocity D, exceeding the velocity of propagation of loading—unloading deformations of the medium, along the top boundary of a strip with a thickness h. The medium filling the strip is modeled by a generalized "plastic gas" [3], and under the load the relation between the pressure p and the volume deformation ε is assumed to be a quadratic polynomial $p = \alpha_1\varepsilon + \alpha_2\varepsilon^2 (dp/d\varepsilon > 0, d^2p/d\varepsilon^2 > 0)$. The slope angle of the unloading branch E of the $p \sim \varepsilon$ diagram is larger than the slope angle of the loading branch, and the profile of the load does not change with the propagation of the waves. In this case, a compression wave with a curvilinear surface Σ (Fig. 1) propagating in the medium with $\xi = x + Dt \geqslant \xi_a$, $\eta = y = h$ is reflected from the rigid boundary in the form of a shock wave with the surface Σ_0, in front of which an elastic wave of a weak discontinuity, as a characteristic of the negative direction, is emitted with a high velocity $c_p = \sqrt{E/\rho_0}$. Because of the propagation and interaction of waves with the boundaries of the strip corresponding perturbed regions 1-4 (Fig. 1) appear. We shall solve the problem for the regions 1-3. If the solution of the problem is constructed in region 1 by the inverse method with a given propagation velocity of the front Σ $d\eta/d\xi = \mathrm{tg}\,\alpha(\xi) = R_1 - R_2\xi$, where R_1, R_2 are fixed constant quantities, then based on [3] for the velocities $u_1(\xi, \eta)$, $v_1(\xi, \eta)$ and the pressure $p_1(\xi, \eta)$ we obtain

$$u_1(\xi, \eta) = -\frac{D}{2\mu}\sum_{i=1}^{2}\frac{(-1)^{i+1}\,\mathrm{tg}\,\alpha[F_i(\xi \mp \mu\eta)]}{\{1 + \mathrm{tg}^2\alpha[F_i(\xi \mp \mu\eta)]\}^2}\{1 \mp \mu\,\mathrm{tg}\,\alpha[F_i(\xi \mp \mu\eta)]\}\Phi_i(\xi \mp \mu\eta), \tag{1.1}$$

$$v_1(\xi, \eta) = \frac{D}{2}\sum_{i=1}^{2}\frac{\mathrm{tg}\,\alpha[F_i(\xi \mp \mu\eta)]}{\{1 + \mathrm{tg}^2\alpha[F_i(\xi \mp \mu\eta)]\}^2}\{1 \mp \mu\,\mathrm{tg}\,\alpha[F_i(\xi \mp \mu\eta)]\}\Phi_i(\xi \mp \mu\eta);$$

$$p_1(\xi, \eta) = -\rho_0 D u(\xi, \eta),\ \Phi_i(z_i) = \left(\frac{\rho_0 D^2}{\alpha_2} - \frac{\alpha_1}{\alpha_2}\right)\mathrm{tg}^2\,\alpha[F_i(z_i)] - \frac{\alpha_1}{\alpha_2}. \tag{1.2}$$

Here $F_i(z_i)$ is the root of the equation $\xi \mp \mu\eta(\xi) = z_i$ with respect to ξ, and in the case i = 1 the upper sign is used.

To find the solution of the problem in region 2, like in [2], we assume that the pressure in front of the reflected wave is the same as in the corresponding points of the incident wave. This means that on different horizontal levels (η = const) $p_2(\xi, \eta)$ on the line AD on the side of region 2 equals $p_1(\xi)$ on the front of the incident wave OA (see Fig. 1). It is assumed at the same time that the front Σ_0 is a slightly curved surface. For a half-plane this is confirmed by the results of [3, 4]. For this reason, all conditions of the problem on the surface Σ_0 are satisfied approximately with respect to its initial

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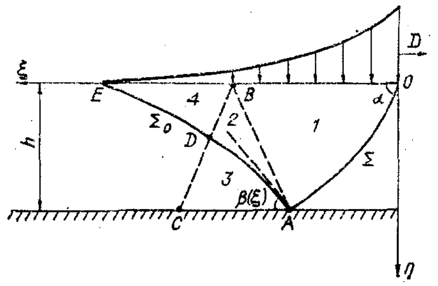

Fig. 1

form, i.e., when $\eta \approx h - \operatorname{tg} \beta_0 (\xi - \xi_a)$. Thus

$$p_2(\xi, \eta) = p_1(\xi) \quad \text{at} \quad \eta \approx h - \operatorname{tg} \beta_0(\xi - \xi_a). \tag{1.3}$$

In addition on the characteristic $\xi + \mu\eta = \xi_\alpha + \mu h$ the conditions

$$u_2(\xi, \eta) = u_1(\xi), \; v_2(\xi, \eta) = v_1(\xi) \tag{1.4}$$

hold. Since the motion of the medium relative to the velocity potential ϕ_2 is described by the wave equation [3], in the region 2

$$\varphi_2(\xi, \eta) = f_1(\xi - \mu\eta) + f_2(\xi + \mu\eta). \tag{1.5}$$

Substituting (1.5) into (1.3) and (1.4), we obtain

$$f_1'(z) = - f_2'(\xi_a + \mu h) - \frac{D}{2\mu} \left\{ \frac{\operatorname{tg} \alpha (F_1(z))}{[1 + \operatorname{tg}^2 \alpha (F_1(z))]^2} \right. \tag{1.6}$$

$$[1 + \mu \operatorname{tg} \alpha (F_1(z))] \Phi_1(z) - \frac{\operatorname{tg} \alpha (F_2(\xi_a + \mu h))}{[1 + \operatorname{tg}^2 \alpha (F_2(\xi_a + \mu h))]^2} \times$$

$$\left. \times [1 - \mu \operatorname{tg} \alpha (F_2(\xi_a + \mu h))] \Phi_2(\xi_a + \mu h) \right\},$$

$$f_2'(z) = - f_1'(\varkappa) - D \frac{\operatorname{tg}^2 \alpha (\xi_1(\nu))}{[1 + \operatorname{tg}^2 \alpha (\xi_1(\nu))]} \left\{ \frac{\rho_0 D^2}{\alpha_2} \frac{\operatorname{tg}^2 \alpha (\xi_1(\nu))}{[1 + \operatorname{tg}^2 \alpha (\xi_1(\nu))]} - \frac{\alpha_1}{\alpha_2} \right\},$$

where

$$\varkappa = \frac{z - \mu (1 + \lambda_0)(h + \operatorname{tg} \beta_0 \xi_a)}{\lambda_0}, \quad \nu = \frac{z - \mu (h + \operatorname{tg} \beta_0 \xi_a)}{(1 - \mu \operatorname{tg} \beta_0)}, \tag{1.7}$$

$$\lambda_0 = \frac{(1 - \mu \operatorname{tg} \beta_0)}{(1 + \mu \operatorname{tg} \beta_0)}, \quad \xi_a = \frac{R_1 - \sqrt{R_1^2 - 2hR_2}}{R_2}, \quad \mu^2 = \frac{D^2}{c_p^2} - 1,$$

$$\xi_1(\xi) = \frac{R_1 - \sqrt{R_1^2 - 2R_2 [h - \operatorname{tg} \beta_0 (\xi - \xi_a)]}}{R_2}$$

(the prime indicates differentiation with respect to the argument). In the region 3 the solution of the problem, like in region 2, is expressed by d'Alembert's formula

$$\varphi_3(\xi, \eta) = f_3(\xi - \mu\eta) + f_4(\xi + \mu\eta). \tag{1.8}$$

At the same time the problem in the region 3 has the following boundary conditions: on the front of the reflected wave Σ_0

$$\rho_2^* (a_0 - v_{2n}^*) = \rho_3^* (a_0 - v_{3n}^*), \tag{1.9}$$

$$\rho_2^* (a_0 - v_{2n}^*)(v_{2n}^* - v_{3n}^*) = p_2^* - p_3^*,$$

$$v_{2\tau}^* = v_{3\tau}^*, \; a_0 = D \sin \beta (\xi),$$

$$p_j^* = \alpha_1 \varepsilon_j^* + \alpha_2 \varepsilon_j^{*2}, \; \varepsilon_j^* = 1 - \frac{\rho_0}{\rho_j^*}, \quad j = 2, 3;$$

on the rigid foundation of the strip with $\eta = h$, $\xi_a \leqslant \xi \leqslant \xi_c$,

$$\partial\varphi_3/\partial\eta = 0, \tag{1.10}$$

where

$$v_n = -\frac{\partial\varphi}{\partial\xi}\sin\beta - \frac{\partial\varphi}{\partial\eta}\cos\beta,\quad v_\tau = -\frac{\partial\varphi}{\partial\xi}\cos\beta + \frac{\partial\varphi}{\partial\eta}\sin\beta$$

are the normal and tangential components of the mass velocity of the medium relative to the front Σ_0; $\beta(\xi)$ is the slope angle of the front of the reflected wave with the axis $O\xi$ ($\beta_0 = \beta(0)$); the parameters of the medium, referred to the front Σ_0, are denoted by an asterisk; and a_0 is the velocity of propagation of the front of the reflected wave.

Setting to a first approximation $\beta(\xi) \approx \beta_0$, we write the third equation of (1.9) with $\eta \approx h - \operatorname{tg}\beta_0(\xi - \xi_a)$ as

$$\left(\frac{\partial\varphi_2}{\partial\xi} - \frac{\partial\varphi_3}{\partial\xi}\right) = \operatorname{tg}\beta_0\left(\frac{\partial\varphi_2}{\partial\eta} - \frac{\partial\varphi_3}{\partial\eta}\right). \tag{1.11}$$

If we take into account the fact that $a_0 \gg v_{2n}^*$ and $\rho_2^* \approx \rho_0$, then the first and third equations in (1.9) assume the form

$$-\rho_0 D(\partial\varphi_2/\partial\xi - \partial\varphi_3/\partial\xi) = p_2^* - p_3^*; \tag{1.12}$$

$$\operatorname{tg}\beta(\xi) = \frac{\left[\left(\frac{\partial\varphi_2}{\partial\eta} - \frac{\partial\varphi_3}{\partial\eta}\right) + \varepsilon_2^*\frac{\partial\varphi_3}{\partial\eta} - \varepsilon_3^*\frac{\partial\varphi_2}{\partial\eta}\right]}{\left[D(\varepsilon_3^* - \varepsilon_2^*) - \left(\frac{\partial\varphi_2}{\partial\xi} - \frac{\partial\varphi_3}{\partial\xi}\right) - \varepsilon_2^*\frac{\partial\varphi_3}{\partial\xi} + \varepsilon_3^*\frac{\partial\varphi_2}{\partial\xi}\right]}. \tag{1.13}$$

We note that $\operatorname{tg}\beta_0$ is determined from the condition (1.9) and (1.10) at $\xi = \xi_a$ (see Fig. 1). Using (1.10) and (1.11), from (1.8) we obtain

$$f_3'(z) = f_4'(z + 2\mu\eta); \tag{1.14}$$

$$f_4'(z) + \lambda_0 f_4'(\lambda_0 z + k_0) = \frac{F\left(\frac{z - \mu h_a}{1 + \mu\operatorname{tg}\beta_0}\right)}{(1 + \mu\operatorname{tg}\beta_0)}, \tag{1.15}$$

where $k_0 = \mu[(1 - \lambda_0)h + (1 + \lambda_0)\operatorname{tg}\beta_0\xi_a]$; $h_a = h - \operatorname{tg}\beta_0\xi_a$;

$$F(z) = (1 + \mu\operatorname{tg}\beta_0) f_1'[(1 + \mu\operatorname{tg}\beta_0)\xi - \mu(h + \operatorname{tg}\beta_0\xi_a)] + (1 - \mu\operatorname{tg}\beta_0) f_2'[(1 - \mu\operatorname{tg}\beta_0)\xi + \mu(h + \operatorname{tg}\beta_0\xi_a)].$$

Solving the functional equation (1.15) by the method of successive approximations, we obtain the following recurrence relation:

$$f_4'(z) = \frac{1}{(1 + \mu\operatorname{tg}\beta_0)}\left\{F\left[\frac{z - \mu(h - \operatorname{tg}\beta_0\xi_a)}{1 + \mu\operatorname{tg}\beta_0}\right] + \sum_{n=1}^{\infty}(-\lambda_0)^n F\left[\frac{\lambda_0^n z - \mu h_a + k_0(\lambda_0^n - 1)/(\lambda_0 - 1)}{1 + \mu\operatorname{tg}\beta_0}\right]\right\}. \tag{1.16}$$

The series (1.16) converges for $\lambda_0 < 1$ (the radius of convergence is easily established in the course of the calculations). Thus from (1.8), taking into account (1.14) and (1.16), we determine the velocity field $u_3 = \partial\varphi_3/\partial\xi$, $v_3 = \partial\varphi_3/\partial\eta$ of the medium in the region 3. The formulas (1.12) and (1.13) also enable determining p_3^* and $\operatorname{tg}\beta(\xi)$. Thus, using the formula $p_3(\xi, \eta) = p_3^* + E(\varepsilon_3 - \varepsilon_3^*)$ (in the region of unloading of the medium the region 3 is of this form), we find the pressure field in it, in particular, on the rigid foundation of the strip. Therefore, the problem in region 3 is completely solved.

For the specific structure of the medium [3] calculations were performed on a computer for

$$\alpha_1 = 12{,}127\cdot 10^2,\ \alpha_2 = 58{,}73\cdot 10^3,\ E = 90{,}16\cdot 10^2,$$
$$\rho_0 = 200\ \text{kg}\cdot\text{sec}^2/\text{m}^4,\quad p_0 = 105\ \text{kg}/\text{cm}^2, \tag{1.17}$$
$$D = 340\sqrt{1 + 0{,}83 p_0},\ R_1 = \operatorname{tg}\alpha_0 = 0{,}1255,\ R_2 = 0{,}86\cdot 10^{-3},\ h = 1{,}2\ \text{m}.$$

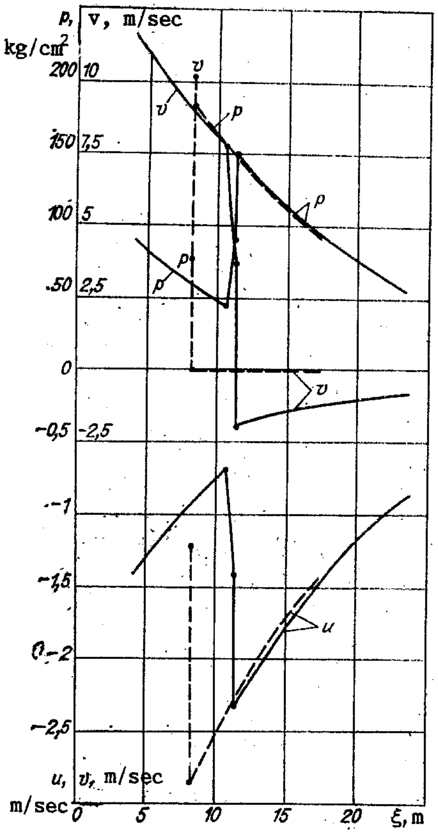

Fig. 2

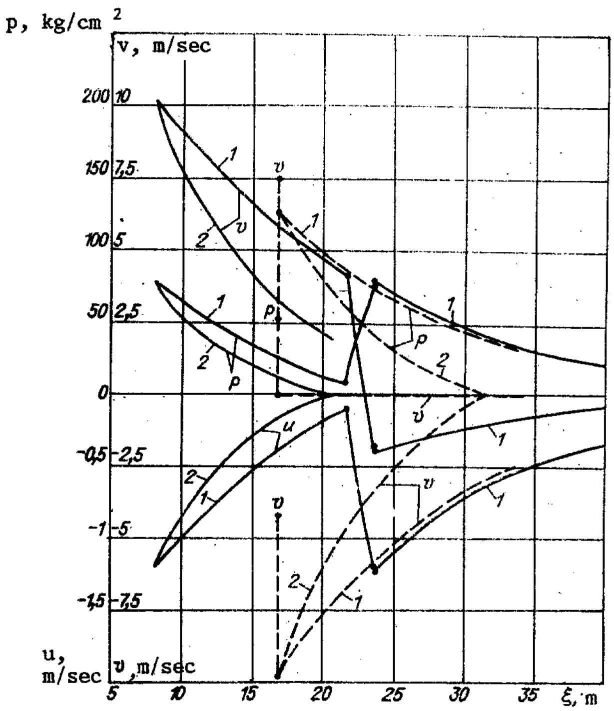

Fig. 3

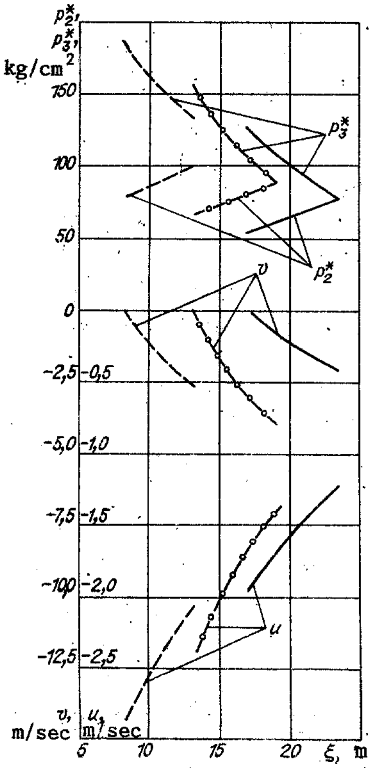

Fig. 4

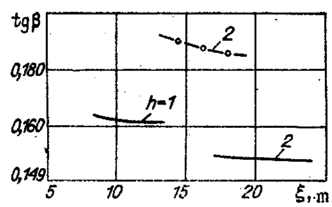

Fig. 5

The results of the calculations are presented in Figs. 2-5 for the pressure, mass velocity, and tgβ (ξ) and a function of ξ in the sections η = h/2 (solid lines), h (broken lines), and along the front of the reflected wave Σ_0. In Fig. 2 it is evident that the parameters p, u, and v in regions 1 and 3 depending on ξ decrease in absolute magnitude in a nonlinear manner. In region 2 p and u increase linearly, and v decreases linearly and changes sign. These parameters reach their maximum values on the corresponding points of the front of the reflected wave on the side of region 3, and their values decrease with increasing thickness of the strip (Figs. 2 and 3, h = 1 and 2 m). In Fig. 3 the curves 1 correspond to the case (1.17) with h = 2 m. In comparing the numerical results it was found that as the coefficients α_1 and α_2 increase the values of the parameters p, u, and v also increase. When Young's modulus E decreases, all parameters of the medium and the time over which they act on the strip correspondingly decrease (Fig. 3, curves 2). Analysis of the curves in Fig. 4 shows that the pressure $p_3^*(p_2^*)$ on the side of the region 3 (2) along the front of the reflected wave depending on ξ gradually decreases (increases), and the vertical (horizontal) component of the velocity of the medium increases (decreases). At the same time, when ξ = 23.6 the reflected wave vanishes. In addition, Fig. 4 also shows for comparison the calculations for $\alpha_1 = 12.127 \cdot 10^2$, $\alpha_2 = 58.73 \cdot 10^3$ with h = 2 m (solid lines) and h = 1 m (broken lines), and for $\alpha_1 = 24.254 \cdot 10^2$, $\alpha_2 = 117.46 \cdot 10^3$ with h = 2 m (lines with circles). Examination of the curve tg $\beta(\xi)$ (Fig. 5) shows that it slowly decreases as ξ increases and, therefore, the front of the reflected wave is a slightly curved surface which is concave with respect to the $O\xi$ axis (see Fig. 1). However, the change in tg $\beta(\xi)$ in the range of ξ studied equals approximately 2-4% of its initial value at the point $\xi = \xi_\alpha$, $\eta = h$.

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